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FORMATION OF A WEAKLY OSCILLATING HIGH-POWER FLOW OF RELATIVISTIC ELECTRONS WITH STRONG MAGNETIC COMPRESSION

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The use of high-power beams of relativistic electrons formed in injectors with thermionic (versus burst) emission is now being advanced [1-3] as the most expedient means of obtaining powerful shf radiation that is stable over microsecond (or longer) time intervals. Such relativistic electron beams (REB), extending over tens of centimers, can attain densities on the order of 10^3 A/cm^2 (with minimum possible transverse velocities) only if the flow is magnetically compressed (in addition to the electrostatic compression, which is effective on the initial sections of the beam [1, 2]). The energy of initially small transverse motions will increase by a factor of 100 or more, the increase being proportional to the degree of compression. If the match between the configurations of the magnetic and electron flows were close to ideal, the quality of the beam that is formed could be satisfactorily characterized by its thermal emittance [1]. When the beam travels through a uniform magnetic field $\mathbf{H}_0$, thermal emittance is only 1.5-2 times greater than the Brillouin value [2] (relative to the density in the crossover of a diode beam) and is the only characteristic needed. However, due to the differences which exist in practice in the configurations of the magnetic and electron flows, the transverse oscillations have a considerably greater influence on the beam than do thermal effects. Thus, it is necessary to study the evolution of transverse motions on all of the initial sections of an electron-optics system (EOS) based on the modified spherical Pierce beam [3] (the magnetic field $\mathbf{H}$ in the resulting REB being an order greater than the Brillouin field). For example, this approach is necessary for several types of shortwave shf generators, including cyclotron-resonance masers (CRM) [4]*. In the quasilinear REB that is formed, small transverse pulsations and drift velocities should be limited by the conditions attached to the radiation mechanism.

The above problem is solved here by selecting the configuration of the field $\mathbf{H}$ in an EOS designed for a CRM with a microperveance of 1.4 and an anode voltage of 300 kV. The unit forms a REB with a power up to 70 MW and with the required diameter of 6-7 mm in a field $\mathbf{H}$ increasing to 7 kOe (where the Brillouin radius equals 0.6 mm); compression on the corresponding initial section relative to the cathode is about 300; when further smooth compression of the beam is necessary in the increasing field $\mathbf{H}$, the adiabatic approximation [5] can be used to calculate the transverse motions with sufficient accuracy and relative ease without allowance for the natural fields of the beam. On the theoretical section being examined, the root-mean-square value of the transverse velocity of the electrons relative to the longitudinal velocity must not exceed 5%. According to estimates, the corresponding relation for thermal transverse velocity will be about 1% with a compression of 300.

For the base configuration of the beam, we took a diode system (Fig. 1) similar to that described in [2] but having a somewhat greater microperveance (1.4 instead of 0.88 – in order to increase the current of the REB) and a correspondingly lower degree of compression (100 versus 400). The latter was achieved by a small shift of the slightly enlarged anode opening toward the cathode. There was almost no change in the gap between the bean and the anode in this case. The diameter of the cathode was 12 cm, the radius of its sphere was 10 cm, the diameter of the REB in the crossover without a magnetic field (at $z = 17$ cm) was 12 mm, and the current of the REB was 220 A at a voltage of 300 kV. The maximum strength of the electric field on the projection on the cathode surface was 105 kV/cm, while it was twice as great on the anode surface. Under conditions of electrostatic compression, such a beam only approximates the requirements established with respect to current

*Among the more attractive alternative methods of forming REBs for CRMs are those which first involve the formation of a thin straight beam. Rotation is then imparted to the electrons in nonadiabatic pumping systems.

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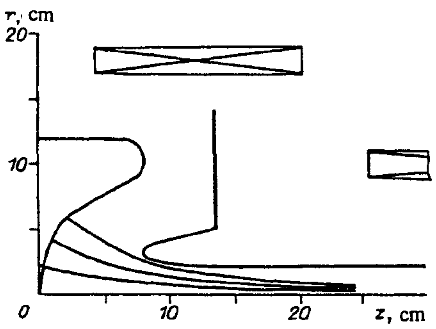

Fig. 1

density in the vicinity of the crossover (where the high quality of the beam is shown by the nearly linear phase characteristic). However, since the beam is characterized by a high degree of nonequilibrium in the crossover when compressed, there is a question as to the feasibility of magnetically tracking the flow (and, if necessary, providing additional magnetic compression) while minimizing lateral oscillations of the path due to deviation of individual stream tubes in the beam from transverse equilibrium.

We choose the above-mentioned EOS (Fig. 1) to numerically model the formation of a REB with magnetic compression. The modeling was continued to distance z along the axis at which the field interaction of the electrons could be ignored, since the evolution of transverse oscillations in a strong magnetic field occurs by an adiabatic law [5]. We used a paraxial approximation (with a specified increasing field $H_z(z)$ on the axis) to perform the calculations for the initial stage. These calculations showed that effective magnetic compression is in fact possible, and they allowed us to establish the required law of change in $H_z(z)$. To perform further calculations with the requisite accuracy, we needed to determine the magnetic field throughout the diode region – including sections farther from the axis. Such a computation requires us to assume the existence of a system of solenoids and find the field **H** from the currents in these solenoids. We choose a system of two solenoids (see Fig. 1): a main solenoid (length 56 cm to z = 81 cm) and a correcting solenoid (closer to the cathode). This allowed us to obtain a magnetic field with force lines that did not differ greatly from the paths of the electrons in the EOS. The force line of the field **H** from the edge of the emitter had a radial coordinate of 3 mm at the center of the main solenoid. With allowance for the inevitable but slight (about 1%) expansion of the REB due to azimuthal drift (with allowance for the intrinsic fields of the REB and the Bush theorem), this corresponded to matching of the required configurations of the beam and magnetic field on the boundary sections of the theoretical region. It should be noted that the thermal expansion of the beam was one to two orders lower than the expansion due to the azimuthal drift.

We performed such a numerical modeling of the REB and optimized the configuration of the field **H** with the use of the algorithms in [6], which were realized in the programs EPOS [7] and EDS-2 [8]. Allowance was made for the intrinsic fields of the beam, except for H_z (diamagnetism). The latter is negligible in such systems, being estimated as hundredths of a percent. The calculations were performed for the region from the cathode to the section z = 24 cm, where the beam was highly magnetized (the Brillouin ratio of the density of the REB and the magnetic field was somewhat to the right of the anode opening the considerably to the left of the crossover in the absence of the field **H**).

In the absence of an external field, the phase portrait of the beam is characterized by the position of representive points (theoretical stream tubes) on the plane $\varepsilon_r \equiv dr/dz$, r, and the effective cross sectional area of the phase volume is the emittance. In a beam with an external **H**, regular rotations with deviations from the trajectories of the "leading" magnetic surface (which coincide with the trajectories on the cathode) make it necessary to account for the complete transverse motions on different projections of the phase space, including the plane $\varepsilon_\theta \equiv r d\theta/dz$, r. We will not use the concept of emittance here, since it is not convenient to describe oscillations and regular drift (at least until the beam becomes highly distorted). In fact, emittance is equal to zero and does not characterize the intensity of transverse motion at all in cases involving cophasal, transversely similar oscillations. We will characterize the quality of a beam with regular transverse motions and gradual distortion by using the rms ratio of transverse velocity to longitudinal velocity:

$$<\varepsilon_\perp> = \left[\frac{\sum (v_\perp/v_{zi})^2 I_i}{\sum I_i}\right]^{1/2} \equiv \left[\frac{\sum \varepsilon_{\perp i}^2 I_i}{\sum I_i}\right]^{1/2} \quad (1)$$

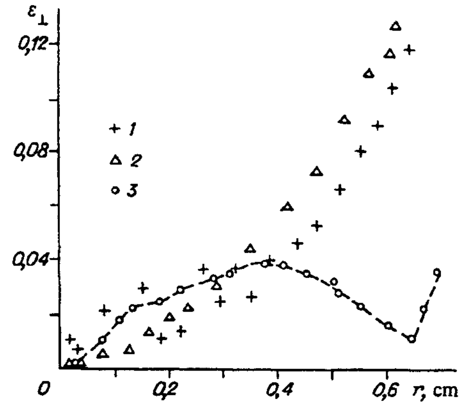

Fig. 2

Here, I_i is the current in the tube; summation is performed over all of the stream tubes; $\varepsilon_{\perp i} = (\varepsilon_{ri}^2 + \varepsilon_{\theta i}^2)^{1/2}$.

In cases when a magnetic field tracks the flow of electrons from the cathode and its configuration is close to the configuration of the flow (and the transverse velocities are low), the radial phase portraits ε_r, r allow us to follow the smooth transition from initially laminar flow to subsequent distortion of the electron beam. However, as noted above, the characteristic of complete transverse motion is more valuable for a number of applications. This characteristic is determined by the position of the stream tubes on the plane $\varepsilon_\perp$, r in the sections z = const, as well as by the rms value $<\varepsilon_\perp>$ according to Eq. (1).

Figure 1 shows the trajectories obtained by a modeling performed with the EPOS program [7] for the initial values of the solenoid fields (chosen in accordance with the method described above). With respect to an axial component H_z, the compression coefficient $\alpha = 288$. The field **H** was substantially nonuniform with respect to the radial coordinate, especially in the neighborhood of the cathode: $|\mathbf{H}|$ appreciably exceeded H_z on the periphery of the electron flow (on the edge of the emitter, by a factor of 1.4). The angular divergence of magnetic surfaces from the trajectories increased toward the periphery due to the difference in the configurations of the electron flow in the diode and the field **H**. This in turn led to significant oscillations near the outside edge of the beam. Points 1 in Fig. 2 represent the slopes of $\varepsilon_\perp(r) = v_\perp/v_z$ of the stream tubes in the section z = 23 cm (phase portrait of the beam), where $|\mathbf{H}| = 2.55$ kOe (the rms value $<\varepsilon_\perp> = 0.063$, the maximum value $\varepsilon_\perp^{max} = 0.116$). Points 2 in Fig. 2 represent the results of similar calculations performed with program EDS-2 [8]. With small differences in the position of individual tubes, these results show nearly the same maximum and rms deviations. The values of total current calculated with this program are 223 and 208 A, respectively. Figure 3 shows the current distributions on the cathode in relation to the length of the arc l. Figure 4 shows the same at z = 23 cm. Their qualitative agreement and an analysis of the trajectories shows that the beam is close to being laminar on this (the initial) section. Recalculation of $<\varepsilon_\perp>$ in accordance with adiabatic theory for the region where $H_z = 7$ kOe gives us $<\varepsilon_\perp> = 0.092$, which is nearly twice as great as the initial requirement for beam quality.

Subsequent modeling was done to optimize the magnetic system in order to minimize oscillations in the REB (the slight azimuthal drift produced by the intrinsic fields of the beam, corresponding to $\varepsilon_\theta \lesssim 0.01$, could not be eliminated). To reduce $<\varepsilon_\perp>$, it is necessary to reduce the angular divergence of the trajectories in the region of the cathode and the field **H** for as much of the beam as possible. This can be done by changing the current in the correcting solenoid, as well as its axial displacement. Figure 5 shows the dependence of transverse velocity $v_\perp^{max}$ (in relative units) at z = 23 cm on the field of the correcting coil H_1 (for the sake of definiteness, we mean the field at the center of the coil). Transverse velocity has a deep maximum at H_1, this maximum amounting to 0.96 of the initial value. The presence of the maximum first of all shows that the choice of magnetic system was a proper one and, secondly, that the oscillatory deviations from equilibrium in the REB are highly sensitive to the configuration of the magnetic field. This sensitivity stems from the fact that the resulting field **H** is comprised of the oppositely directed fields of the main and correcting solenoids, which are close in the strength on the initial sections. However, the value of $v_\perp^{max}$ remains substantial at the optimum value of H_1, which is related to the difficulty of ideally matching the configurations of the REB and field **H**. Figure 6 shows the electron trajectories and magnetic lines of force on the "diode" beam-forming section for the optimum variant $H_1 = 0.96H_1^{ini}$. The phase portrait at z = 23 cm is shown in Fig. 2 (points 3). The maximum oscillations and, thus, the maximum of $\varepsilon_\perp$ now occur roughly at the middle of the beam,

*We used the stream tube method [6] in the modeling: the flow was divided into 60 (EPOS) or 70 (EDS-2) tubes.

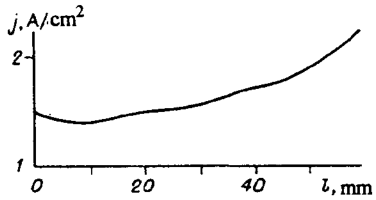

Fig. 3

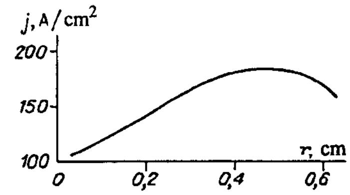

Fig. 4

rather than on the periphery. In this case, $\varepsilon_{\perp}^{max} = 0.04$, while the rms value $<\varepsilon_{\perp}> = 0.025$. A recalculation performed for the outlet of the EOS (where H = 7 kOe) gave $\varepsilon_{\perp}^{max} = 0.06$, $<\varepsilon_{\perp}> = 0.038$. However, the beam expands somewhat (by 20% relative to the initial variant) due to the some concentration of the magnetic surfaces at the cathode with a decrease in H_1). The magnitude of the initial pulsations is consistent with the initial requirements.

To optimize the magnetic system, we also performed calculations of REB formation with displacement of the correcting coil away from the cathode. We also calculated results for different points of the coil at each position. Values of $<\varepsilon_{\perp}>\cdot 10^2$ are shown in Table 1, where Δz is the displacement of the coil; $\Delta I/I$ is the relative change in current in the coil. It is evident that for different Δz it is possible to choose the current so as to obtain roughly the same minimum value $<\varepsilon_{\perp}> = 0.025$ at z = 23 cm and, thus, obtain $<\varepsilon_{\perp}> \approx 0.04$ at the outlet of the EOS. In this case, the rms of the radius of the beam $<r_b>$ $\approx$ 2.8 mm. This satisfies the initial requirements for beam quality with a margin of error.

As noted above, we choose a well-known gun [1] as our base design. The highly accurate algorithms described in [1, 2], realized in the application package SAM [9], make it possible to reliably model the quasi-laminar structure of the beam in EOSs similar to those studied in [1] without an external magnetic field. The structure of the beam can be modeled up to the anode opening. Here, we present results of calculations performed for a problem which is considerably more difficult from a computational viewpoint. First of all, we increased the length of the theoretical section relative to [1, 2] by a factor of 1.9: from 1.6d to 3d (where d is the length of the gap between the anode and cathode). Secondly, we introduced a magnetic focusing system which established an increasing external magnetic field with a compression of about 300 (7 kOe at the outlet). The external field produced a beam with a substantially nonlaminar structure, and it imposed additional limitations on the time step in the integration of the equations of motion for individual particles (these limitations being connected with the accuracy of calculation of the Larmor precession).

To improve the accuracy of the results, we performed calculations for guns with known characteristics [1, 10, 11] and parameters similar to the parameters of the gun in our investigation. No magnetic field was present in this case.

Agreement with the numerical results in [1, 10, 11] was obtained only after we refined the algorithms used to model the dynamics of the beam. In particular, as in [12], we improved the accuracy of calculation of the strength of the natural electrostatic field near the boundary and axis of the beam.

No calculations have been performed by Russian investigators for guns of the given type with allowance for an external magnetic field along a symmetry axis of order 3d. Thus, to make comparisons and improve the reliability of the numerical results, we performed independent calculations using the EDS-2 and EPOS programs. These programs take different approaches to computation of the electrostatic potential: the finite-difference method is used in EDS-2, while the EPOS employs the method of integral equations and secondary charges. The calculations yielded similar results, which are shown in Table 1.

The calculations for the EOS shown in Fig. 1 were performed by the EDS-2 on an ES 1066 computer with the following main computing parameters: 100 and 251 nodes along the r and z axes, respectively; an increase in the number of nodes near the cathode (total number 25,100); number of stream tubes 70. The computation was ended after the twenty-first element iteration, so that the cathode current on the last two iterations was 208.28 and 208.27 A. It took 10 min to calculate the components H_r and H_z of the magnetic field of the solenoids at the node of the difference grid corresponding +28 min in the 21st external iteration. Despite the refinement of the grid representation of the beam's cross section, only six nodes fell within the crossover region. This in particular illustrates the difficulty of modeling the microstructure of the beam in this region. This in particular illustrates the difficulty of modeling the microstructure of the beam in this region.

We make two important observations here: 1) the seeming possibility of a reduction in compression (and the size of the cathode) due to low current density on the cathode (see Fig. 3) cannot occur in practice due to a corresponding reduction in the dielectric strength of the system: in the proposed EOS (see Fig. 1), the maximum electric field on the non-emitting projection of the cathode surface is about 130 kW/cm; 2) a decrease in transverse oscillation can be expected with a decrease in the angular dimension of the emitter, i.e. with a transition to a beam with a lower perveance and the approximation of para-

TABLE 1

$\Delta I/I$, %	Δz, cm				
	0.5	1.0	1.25	1.5	2.0
	$\langle\varepsilon_\perp\rangle\cdot 10^2$				
3	6,5	2,44		5,57	
6	11,0	5,13	2,41	2,74	7,0
9	15,0	9,0		5,1	

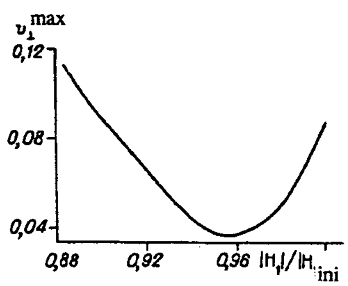

Fig. 5

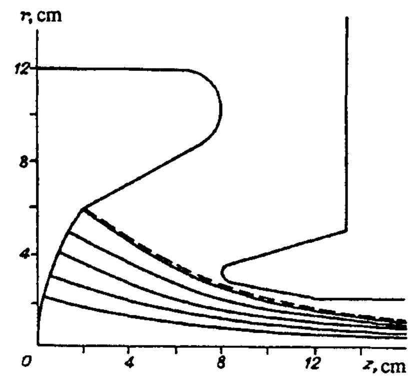

Fig. 6

paraxial beam-forming conditions (preliminary calculations show that the regular transverse velocities approach the thermal velocities when microperveance is 0.6 (current 90 A)).

We end by making the following conclusions.

To form a REB in thermal injectors with strong magnetic compression so as to obtain a current density on the order of 10^3 A/cm^2 and the parameters needed for shf electronics, it is possible to select a magnetic field configuration in which the flow is smoothly "captured" in the region of the anode opening without serious distortion of the electrostatic compression occurring in the diode part of the EOS. The chosen configuration also permits magnetic tracking with effective compression in a long channel under conditions of heavy magnetization (with feasible limits on electron oscillation).

Oscillatory deviations from equilibrium trajectories in the fully formed REB – almost inevitable because of the noncoincidence of the magnetic surfaces and the stream tubes of the beam – can be also be controlled by proper selection of the field configuration. In the practically important magnetic system of two solenoids, the rms ratio of transverse to longitudinal velocity at the outlet of the EOS can be kept to 4% or less for a beam with a current of 220 A, electron energy of 300 kV, and magnetic compression of about 300 (7 kOe at the outlet). Here, the rms radius of the beam is no greater than 3 mm. Further compression and transport can easily be calculated on the basis of existing adiabatic theory.

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